Student Use of Direct Modeling Strategies on Groups-of-Ten Word Problems

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

This lesson began with a set of three word problems posed to second-grade students as a way to assess students' understanding of place value and their problem solving abilities. The teachers studied the strategies students used to solve the problems as a way to gain insight into the students' understanding of place value. The teachers then designed and implemented a lesson to help students make sense of problems and advance their place value understanding.

Relevant Florida Mathematics Standards

MAFS.2.NBT.1.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

MAFS.2.NBT.1.2 Count within 1000; skip-count by 5s, 10s, and 100s.

MAFS.3.OA.1.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Background Information

In preparation for this lesson, consider familiarizing yourself with the ways in which children think about and solve multiplication and division problems. An introduction to this topic can be found in chapters four and six of *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 2015). Chapter four provides a framework in which student strategies used to solve addition word problems are extended to multiplication word problems through the grouping of countable objects. Chapter six focuses on children's mathematical thinking and problem-solving strategies in relation to multidigit numbers.

The lesson places emphasis on *counting* strategies, especially skip-counting, and organizing objects into groups of ten. An article named *Counting Collections* (Schwerdtfeger and Chan, 2007) offers a good introduction to the importance of guiding children's counting processes to create the basis for more sophisticated number sense, such as counting groups of ten to solve multiplication with base-ten concepts.

Carpenter, T. P, Fennema, E., Franke, M. L, Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction* (Second Edition). Portsmouth, NH. Heinemann.

Schwerdtfeger, J. K. & Chan, A. (2007). Counting collections. *Teaching Children Mathematics*, 13(7), 356—361.

Analyzing Student Thinking

To prepare for the lesson, a group of teachers learning about student strategies conducted one-on-one interviews with students in a second-grade class to improve their understanding of students' current mathematical thinking. In the interviews, the students were asked to solve three word problems:

A. I have 5 boxes with 6 cookies in each box. How many cookies do I have?
B. I have 8 boxes of pencils. There are 10 in each box. How many pencils do I have?

C. The second graders at Orange Grove Elementary School raised $67 to buy books for the children's hospital. If each book cost $10, how many books can they buy?

Before conducting the interviews, the teachers anticipated the various strategies students might use for each of the three problems. They asked themselves "what might a second grader do?" and focused on children's commonly known solution strategies.

During the interviews, the teachers read one problem aloud at a time and allowed the students time to think, model, write, and explain their answers. The students had linking cubes, base-ten blocks, and pencil and paper at their disposal and were encouraged to use whatever tools or methods they wished to solve the problem. A few students who needed translation into Portuguese were paired with teachers who were fluent in Portuguese. The teacher interviewers asked probing questions to improve their understanding of students' mathematical thinking while intentionally avoiding showing children how to solve the problems. The interviewers had the flexibility to change the context of the problems and to lower the numbers if they found the larger numbers were an obstacle to the students' solving the problem. For example, they might change $67 to $23 in problem C if the student became confused or frustrated.

Types of Strategies for Problem A[1]

The student who uses a *direct modeling* strategy generates the solution by representing each number in the problem by means of manipulatives (including fingers) or by drawing a representation with the writing materials. Figure 1 provides an example of a *direct modeling* strategy for Problem A. The student drew five boxes and placed six dots in each box to represent the cookies. She then counted all the dots one by one to find the answer of 30 cookies.

Figure 2 illustrates a variation of *direct modeling*. In this example, the student modeled every box with every cookie and appears to skip count or repeatedly add by sixes rather than to count each cookie individually.

The student who uses a *counting* strategy uses an approach that does not represent all quantities in the problem concretely. For example, a student using a *counting* strategy might skip-count by sixes five times to find the answer—6, 12, 18, 24, 30—without the use of a model. *Counting* strategies involve more abstract thought than a *direct modeling* strategy.

The student who uses a *fact-recall* strategy knows the answer from memory or responds that he or she "just knows" the answer. A student using this strategy would just "know" that 5 groups of 6 is 30. No student used this strategy in this case.

Types of Strategies for Problems B and C

For problems B and C, a different set of categories was used for sorting student strategies. Problems B and C are similar to Problem A in that they

1 The descriptions of strategies presented in this section are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

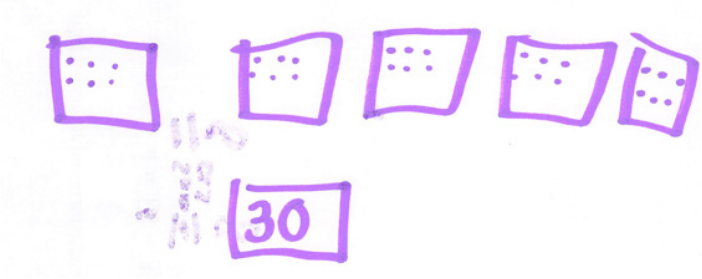

Figure 1. ***Direct modeling*** *strategy for problem A.*

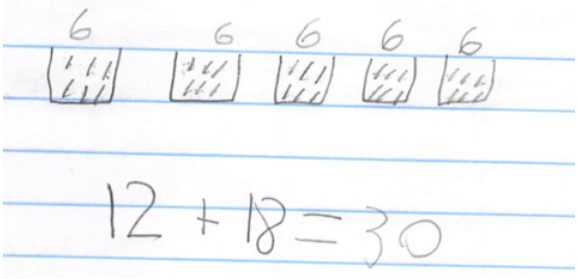

Figure 2. ***Direct modeling*** *with adding on in chunks for problem A.*

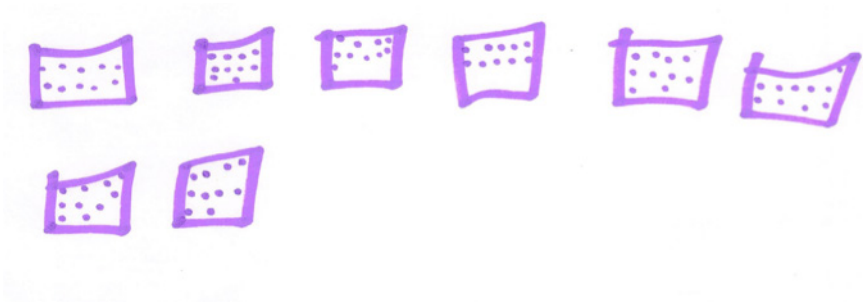

Figure 3. ***Counting by ones*** *student strategy for problem B.*

all involve groups of objects in the story context. Problems B and C are different from Problem A (but similar to one another), because the context of the story involves groups of ten. Place value concepts involve grouping by tens, so those problems are more likely to yield insight into students' understanding of place value.

The student who uses a *counting by ones* strategy represents all the quantities in the problem (e.g., placing ten of something in each of eight groups) and counts by ones to determine how many are there. Figure 3 is an example of work from a student using such a strategy to solve problem B (which involves eight boxes of pencils with ten pencils in each box). The student drew eight boxes and put ten dots in each box. She then counted the dots by ones and reported a total of 80 pencils.

The student who uses a *counting by tens* strategy may use *direct modeling* or *counting* strategies. This category is operationally defined as representing the quantities in the problem and *counting by tens* to determine the solution. A typical example of this strategy is shown in Figure 4, where the student drew a picture of groups of ten dots and then counted the dots by tens to find the total.

The student who uses a direct place value strategy knows how many tens and ones are in the given number and provides the answer without creating a pictorial representation. A typical example of this strategy from Problem B is a student who responded "80, because 8 tens is 80."

Strategies Used by Students in This Class

After the students solved the three problems, the teachers sorted the students into groups based on the strategies they observed students using. Figure 5 displays the charts the teachers used to organize the students by problem, strategies used, and numbers used. Most students interviewed modeled each number, but they differed

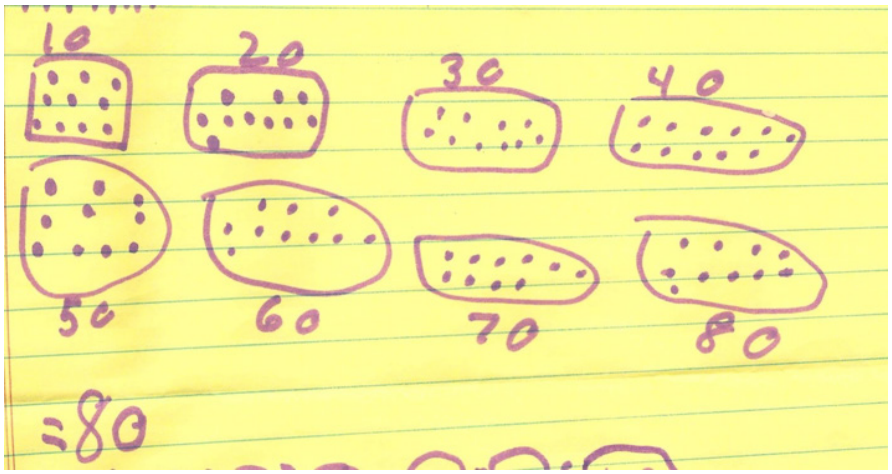

Figure 4. ***Counting by tens*** *student strategy for problem B.*

Problem A			
Could not solve	Counting	Direct modeling	Known facts
Zachary	Louie	Derisah	
Stephie	Stephie	Jassir	
Kahua	Daniel	Kathleen	
Lara	Raphael	Leonardo	
Angelica	Daniel	Alessandra	
		Aiden	
		Sheldon	
		Sarah	
		Isabella	

Problem B			
Could not solve	Counting by ones	Counting by tens	Known facts
Lara	Isabella	Sheldon	Daniel
Angelica	Derisah	Jassir	
Zachary		Kathleen	
Raphael		Leonardo	
		Alessandra	
		Aiden	
		Sheldon	
		Sarah	
		Isabella	

Problem C			
Could not Solve	Counting by ones	Counting by tens	Known facts
Lara		Sheldon	Daniel
Angelica		Louie	Jassir
Zachary		Sarah	Alessandra
Stephie		Leonardo	
Derisah		Aiden	
Isabella		Raphael	
Kathleen			

Figure 5. Student strategies for each of the three interview problems.

in the ways in which they counted and found the answer to the problem. The teachers were happy to notice the variety of strategies already used by students, especially since this was the students' first time working with multidigit numbers in this class. The majority of students used strategies involving groups of ten at least some of the time.

Study of the charts revealed that the largest groups of students used a *direct modeling* strategy for Problem A, a *counting by tens* strategy for Problem B, and a *counting by tens* strategy for problem C (although an equal number of students or could not solve problem C at all). After a discussion centered around place value and why so many students were not able to solve the last problem, the teachers concluded that the students' understanding of place value was not as solid as the results for problem B would suggest.

The teachers also discussed the possibility that story-comprehension issues might have interfered with students' ability to successfully solve problem C. When they asked students to retell the story, they found that some students had a difficult time remembering the story or the numbers in it. Several teachers mentioned they changed the context of the second problem to something more familiar before students were able to solve it.

Setting Learning Goals for the Lesson

The teachers decided to develop a lesson with the goal of deepening the students' understanding of the place value while also focusing on increasing story comprehension. (The teachers agreed it was very important to make sure the students who were not able to solve the three initial word problems were at least able to understand what the problem asked before they attempted to solve the new problem.) They set the following goals for the lesson.

1. *Students will understand and be able to reproduce the story in a word problem.*

2. *Students will be able to skip-count by ten with understanding in the context of solving a story problem.*

3. *Students will use place value understanding to solve a word problem involving groups of ten.*

Planning for the Lesson

After much discussion, the teachers decided to build the lesson around a measurement-division word problem involving groups of ten. This would be a problem type students were struggling to solve in the morning, and the teachers thought continuing that struggle with support from the teachers and peers may provide an opportunity for students to improve their understanding of place value. They formulated the following mathematics problem for the lesson.

> *Mrs. Kennedy has 70 markers. She wants to put her markers in boxes so there are 10 markers in every box. How many boxes can she fill?*

Rationale for the problem selected

The purpose of the chosen problem type was to get more students to count by tens. The teachers deliberated whether they should pose a problem with a multiplication-grouping context or a measurement-division context. Through a discussion,

After some consideration, the teachers decided to use a number of markers that would fill all boxes and leave no markers left over. They chose 70, because it is a large double-digit number that would discourage students from counting by ones and may prompt them to look for more efficient strategies.

This variation would be likely to support interesting class discussion, because students could compare and contrast different students' strategies.

they concluded that the latter would be the more supportive context for the goal, because it draws the attention of the students to separating an amount into groups of ten.

The teachers also noticed that a good number of students in the class already demonstrated strategies involving grouping by tens and then either counted by ones or counted by tens during the interviews. This variation would be likely to support interesting class discussion, because students could compare and contrast different students' strategies.

Strategy for differentiation to meet the needs of all students in the class

The teachers agreed to spend a significant proportion of the lesson time preparing the students to solve the problem to help the large group of students who struggled with story comprehension. This preparation would involve rereading the problem, asking what happened first, asking what happened second, and asking students to reproduce the story. Teachers wanted to make sure all students understood what was being asked and what happened first and second in the story of the problem. Teachers planned to look for students who did not know how to start working on the problem and to ask them questions about the different elements of the story.

On the basis of their prior experience with the children, the teachers determined that situating the problem in a familiar context such as boxes of markers would be helpful. The teachers were also purposeful in choosing numbers that would help foster a growing understanding of groups of ten. In this case, it involved boxes that could hold ten markers. After some consideration, the teachers decided to use a number of markers that would fill all boxes and leave no markers left over. They chose 70, because it is a large double-digit number that would discourage students from *counting by ones* and may prompt them to look for more efficient strategies. They also considered the strategies they would want to share with the whole class, the order in which the strategies would be shared, and what connections to make.

Teachers decided on a tentative order of anticipated strategies that they would want to have shared in class discussion. By structuring the discussion to include strategies along a continuum of complexity, the teachers conjectured that students would be able to participate in the discussion at their current level of understanding. The teachers planned to encourage students to make connections between strategies by displaying two strategies side by side. They also planned to emphasize *counting by tens* by having the entire class

count out loud.

In addition, the teachers developed a possible extension to the problem for those who finished early, using the numbers (82, 10) and (153, 10).

Lesson Plan

The teachers previously developed the following learning goals for this lesson:

1. Students will understand and be able to reproduce the story in a word problem.
2. Students will be able to skip-count by ten with understanding in the context of solving a story problem.
3. Students will use place value understanding to solve a word problem involving groups of ten.

Before the lesson, write down the problem for display to the students. For this portion of the lesson, do not include the numbers in the problem. These will be revealed later during the lesson.

1. Present the Mrs. Kennedy's markers problem without numbers and help students to comprehend the situation in the problem. Display the written problem to the class. Ask students to read the problem silently and signal when they have finished. When they have all finished, ask them to read it again the same way. Ask for a volunteer to read the problem out loud. Next, read the problem out loud again, this time with the students in chorus.

   Mrs. Kennedy has _____ markers. She wants to put her markers in boxes so there are ____ markers in every box. How many boxes can she fill?

2. Ask the students, "What happened in the beginning of the story?", then wait for an answer. Ask "What happens next?" and wait for an answer. Ask students who had demonstrated story comprehension deficiencies during the interviews to find out whether they comprehend the current problem. Next, ask the class "What is the question sentence? What are we trying to find out?" If students still struggle with comprehending the story, ask students "What does Mrs. Kennedy want to do in the story with the markers?" Ask them to share with a partner instead of offering an answer out loud. While students are talking to their shoulder partners, walk among them, listen to their ideas, and join in with probing questions. After a few minutes, when you notice most students have an idea, ask them to share their answers. The expected answer is "she wants to put markers in boxes."

3. Reveal the numbers for the Mrs. Kennedy's markers problem and provide instructions for work time. Tell the students that Mrs. Kennedy wants to put 70 markers in boxes so that every box has ten markers. Tell students that when they have finished with the problem they can challenge themselves to figure out how many boxes could be filled if Mrs. Kennedy had 82 markers and ten were to go in each box. Tell the class that anyone who finds both answers will receive another challenge problem. Give such students the problems on paper, and send them to work on the problem independently at their desks.

4. Circulate and observe students' approaches to solving the problem. For those who use linking cubes, notice who breaks the rods apart and who works with the connected rods.

5. Interact with students to support and extend their thinking.

   a. If a student is struggling to find a starting point, ask scaffolding questions about story comprehension: "Can you tell me something about the story?" or "What does Mrs. Kennedy want to do?"

   b. If the student is using a strategy described in Phase 2, ask students to explain how they are thinking about the problem. Ask "How did you get that?" or "Can you count this for me please?" Refrain from offering immediate feedback beyond acknowledging that you understood the strategy. Consider

using the phrase "I see how you did that" rather than "Good job."

c. For early finishers, provide the new pairs of numbers such as (82, 10) or (153, 10).

6. Identify students who will be asked to share their strategies during whole-class discussion in order to stimulate consideration of specific mathematical ideas. Look for students using *direct modeling* strategies with groups of ten, especially with linking cubes, and those who modeled in writing. Select two or three students who show at least a rudimentary understanding of place value in how they handled the manipulatives, in how they counted, or in how they grouped the model. Keep an eye out for number sentences with repeated addition. Decide on an order in which you want them to share with the rest of the class. Consider beginning with a less sophisticated strategy first and then invite incrementally more sophisticated strategies to share in the discussion, making sure to include at least one strategy involving *counting by tens*. Attempt to select strategies that represent similar ideas with different representations so that students can see the connections between them. An example sequence appears in step number 8 of this lesson plan.

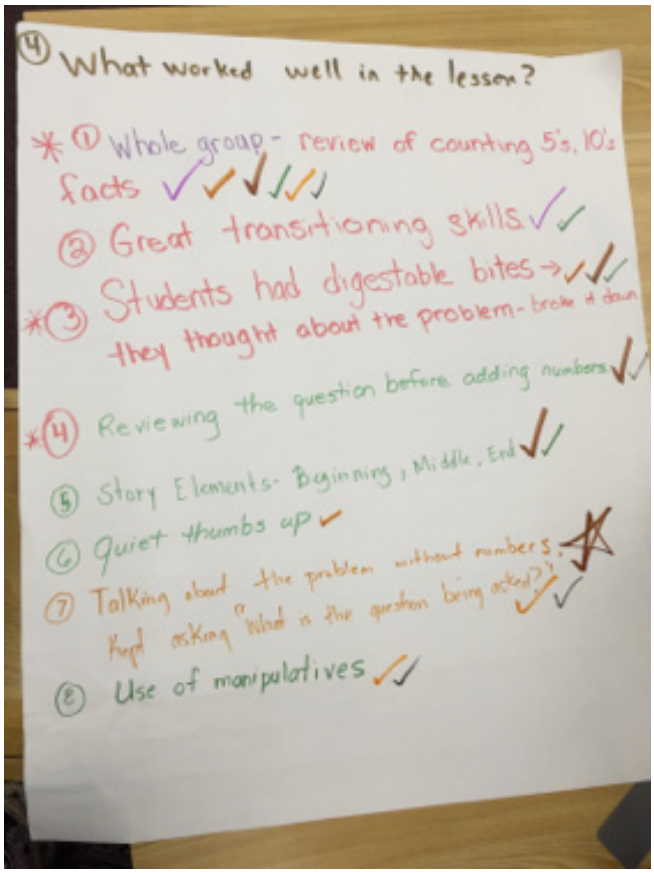

Figure 6. A chart of what the teachers felt worked well during the classroom embedded lesson

7. Gather the whole class for a summarizing discussion.

8. Invite the students you have identified to share their strategies one at a time. Again, consider sequencing strategies in order of least sophisticated to most sophisticated.

   a. Begin by having a student who has used linking cubes connected in rods of ten share his or her thinking. After the student sets up manipulatives so that everyone can see them, ask the class to think what the student was thinking and to share their thoughts quietly with a partner for one minute. Ask the student at the board to explain the strategy used and count to show how he or she found the answer. Ideally, the student will explain that the 70 cubes represent the

markers that are placed ten in each box and will count by ten while touching each rod. Ask another student to come and count the cubes. Do not correct a student who correctly counts by ones. Instead, ask the class whether the two students counted similarly or differently. Make a connection between the two ways of counting the same number of cubes. Ask the students which ones are the markers and which ones are the boxes.

b. The next strategy shared should be a more sophisticated understanding of ten. Ask a student who used drew an enclosure to model each box and wrote the number ten underneath.

c. Ask the class to compare the two strategies that have been shared; what is the same and what is different about the two strategies? You may want to place them side by side to help students compare.

9. End the lesson by ask students to share with their partner one idea that they learned today. As time permits, you may want to have a few share what they learned out loud.

Reflection

What we have learned about the students

The lesson was helpful in getting students to understand what was happening in the story and what actions they needed to model in order to solve the problem. The teachers noticed that having students retell the story encouraged them to visualize it more. Even though a student may only be able to reproduce one part of the story, the whole-class discussion at the beginning of the lesson helped students understand what was happening in the story. The teachers found that skip-counting proved to be a useful way for students to count their models.

Figure 6 includes a full list of things the teachers thought went well in the lesson. The checkmarks are there to indicate that more than one teacher agreed with a statement that was already written.

The teachers noticed that asking students to guess what students did in their strategies was also successful in helping students to make connections between strategies on their own. They also noticed that asking questions about how the strategies were similar and how they differed helped students notice that the groups of markers can be counted in different ways. The discussion time at the end was shortened to allow more time at the beginning for helping the students understand the story, but the change was necessary to ensure that student understood the action presented in the story and were able to use that understanding when they were working individually to solve the problem.

Ideas for the next lesson

The initial numbers chosen for the word problem had students deal only with boxes of markers filled all the way, without extra markers left over. The next number of markers given to students, once they solved the problem, introduced a number that was not a multiple of ten, in this case 82, which offered the additional complication of having to decide what to do with the two markers left over.

Many students did not have time to solve the problems with the higher numbers, so the next steps could be to pose a problem with a measurement-division problem that includes a number that is not a multiple of ten. Solving problems with numbers like that can prompt students to deepen their understanding of the base-ten number system by having to consider what to do with items that do not constitute an entire group. Such a number can also highlight the difference between how many tens and how many ones are in a multidigit number.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**